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Interplay of Spin and Orbital Angular Momentum in the Proton

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Abstract

We derive the consequences of the Myhrer-Thomas explanation of the proton spin problem for the distribution of orbital angular momentum on the valence and sea quarks. After QCD evolution these results are found to be in very good agreement with both recent lattice QCD calculations and the experimental constraints from Hermes and JLab.

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I. INTRODUCTION

There is no more fundamental question concerning the structure of the nucleon than the distribution of spin and orbital angular momentum over its quarks and gluons [1, 2]. This issue has been of enormous topical interest since the European Muon Collaboration (EMC) reported that most of the nucleon spin was not carried as the spin of its quarks and anti-quarks [3]. Over the last 20 years there has been tremendous progress in unravelling this mystery. In particular, it is now known that the missing spin fraction is of order 2/3 [4, 5], rather than 90% and furthermore the contribution from polarized gluons is less than 5% (corresponding to $|\Delta G| < 0.3$ [6, 7, 8, 9, 10, 11]). It was recently shown by Myhrer and Thomas [12] that the modern spin discrepancy can be rather well explained in terms of standard features of the non-perturbative structure of the nucleon, namely relativistic motion of the valence quarks [13], the pion cloud required by chiral symmetry [14] and an exchange current contribution associated with the one-gluon-exchange hyperfine interaction [15].

Here we derive the consequences of the Myhrer-Thomas work for the distribution of orbital angular momentum on the quarks and anti-quarks. These results are then tested against the latest measurements of the Generalized Parton Distributions from Hermes and JLab, as well as lattice QCD. We shall see that once the appropriate connection between the quark model and QCD is made at an appropriately low scale, there is a remarkable degree of consistency between all three determinations. This not only gives us considerable confidence in the physical picture provided by Myhrer and Thomas but it also provides much needed insight into the physical content of the lattice QCD simulations.

The structure of the paper is that we first track where, in the Myhrer-Thomas picture, the missing spin resides as orbital angular momentum on valence quarks and anti-quarks. We then recall that orbital angular momentum is not a renormalization group invariant and argue, following 30 years of similar arguments [16, 17], that the model values should be associated with a very low scale. Solving the QCD evolution equations for the up and down quark angular momenta then leads to the remarkable result that the orbital angular momentum of the up and down quarks cross over around 1 GeV^2, so that at the scale of current experiments or lattice QCD simulations L^d (the orbital angular momentum carried by down *and* anti-down quarks) is positive and greater than L^u, which tends to be negative.

Consider first the relativistic motion of the valence quarks, described (e.g.) by solving the

Dirac equation for a spin up particle in an s-state. The lower component of the corresponding spinor has the quark spin predominantly down (i.e. spin down to spin up in the ratio 2/3:1/3), because the corresponding, p-wave orbital angular momentum is up. Thus the relativistic correction which lowers the quark spin fraction to about 65%, leads to 35% of the proton spin being carried as valence quark orbital angular momentum. If, for simplicity, we start with an SU(6) wavefunction, the $u-d$ components are in the ratio +4/3:-1/3. This is summarized in line 2 of Table 1.

As originally derived by Hogaasen and Myhrer [18], the exchange current correction to spin dependent quantities, such as baryon axial charges and magnetic moments, arising from the widely used one-gluon-exchange hyperfine interaction, is dominated by those diagrams involving excitation of a p-wave anti-quark. The total correction to the spin is $\Sigma \to \Sigma - 3G$, where $G = 0.05$ involves exactly the same matrix elements. (N.B. We follow Hogaasen and Myhrer in using G to denote the product of α_s times the relevant bag model matrix elements. It bears no relation to the gluonic parton distribution or ΔG, which is traditionally used to denote the spin carried by polarized glue.) In this case, the 15% of the proton spin lost to quarks through this mechanism is converted to orbital angular momentum of the p-wave anti-quark. This is summarized in line 3 of Table 1.

The pion cloud of the nucleon required by chiral symmetry [19, 20, 21] leads to a multiplicative correction to the nucleon spin, $Z - \frac{1}{3}P_{\mathrm{N}\pi} + \frac{5}{3}P_{\Delta\pi}$ [41] of order 0.75 to 0.80. For the N π Fock component of the nucleon wavefunction the angular momentum algebra is identical to that of the lower component of the quark spinor mentioned above. That is, the pion tends to have positive (p-wave) orbital angular momentum, while the N spin is down. From the point of view of a deep inelastic probe the pion is (predominantly) a quark-anti-quark pair but since they are coupled to spin zero they contribute nothing to the spin structure function.

The flavor structure of the pion-baryon Fock components needs a little care, for example the dominant N π component is n π^+, so the pion orbital angular momentum in this case is shared by a u-quark and a $\bar{d}$-anti-quark – leading naturally to an excess of $\bar{d}$ quarks in the proton sea [22]. The final distribution of spin and orbital angular momentum, obtained after applying the pionic correction to the relativistic quark model, including the effect of the one-gluon-exchange hyperfine interaction, is shown in the final line of Table 1.

The very clear physical picture evident from Table 1 is that the spin of the proton

TABLE I: Distribution of the fraction of the spin of the nucleon as spin and orbital angular momentum of its constituent quarks at the model (low energy) scale. Successive lines down the table show the result of adding a new effect to all the preceding effects. (Note that for all terms the contributions of both quarks and anti-quarks of a given flavor are included.)

	L^u	L^d	Σ
Non-relativistic	0	0	100
Relativistic	0.46	-0.11	0.65
OGE	0.67	-0.16	0.49
Pion cloud	0.64	-0.03	0.39

resides predominantly as orbital angular momentum of the u (and $\bar{u}$) quarks. In contrast, the d (and $\bar{d}$) quarks carry essentially no orbital angular momentum. The total angular momentum is shared between the u (and $\bar{u}$) quarks, J^u, and the d (and $\bar{d}$) quarks, J^d, in the ratio $J^u : J^d = 0.74 : -0.24$. (Note that there are no strange quarks in the Myhrer-Thomas calculation, so Σ in Table 1 is $\Delta u + \Delta d$. Combining this with $g_A^3 \equiv \Delta u - \Delta d = 1.27$ yields these values. A more sophisticated treatment, including the KN Fock component of the proton wavefunction [23], would lead to a very small non-zero value of Δs [24].)

At first appearance, these results seem to disagree with the first indications from lattice QCD [25, 26], which suggest that L^d tends to be positive, while L^u is negative. One should observe that these calculations were performed at fairly large quark mass and omit disconnected terms, which may carry significant orbital angular momentum [27] and are certainly needed to account for the U(1) axial anomaly. Nevertheless, the apparent discrepancy is of concern.

At this point, we recall the crucial fact that neither the total, nor the orbital angular momentum is renormalization group invariant (RGI) [28]. The lattice QCD values are evaluated at a scale set by the lattice spacing, around 4 GeV^2. On the other hand, we have not identified the scale corresponding to the values derived in our chiral quark model. Indeed, there is no unambiguous way to do so unless the model can be derived rigorously from non-perturbative QCD.

This problem has been considered for more than 30 years [16], driven initially by the

fact that in a typical, valence dominated quark model, the fraction of momentum carried by the valence quarks is near 100%, whereas at 4 GeV^2 the experimentally measured fraction is nearer 35%. Given that QCD evolution implies that the momentum carried by valence quarks is a monotonically decreasing function of the scale, the only place to match a quark model to QCD is at a low scale, Q_0. Early studies within the bag model found this scale to be considerably less than 1 GeV [17].

Over the last decade, this idea has been used with remarkable success to describe the data from HERA, over an enormous range of x and Q^2, starting from a valence dominated set of input parton distributions at a scale of order 0.4 GeV [29]. A similar scale is needed to match parton distributions calculated in various modern quark models to experimental data [30]. We note that the comparison between theory and experiment after QCD evolution is not very sensitive to the order of perturbation theory at which one works. However, what does change is the unphysical starting scale. For this reason we present results here at leading order - which also avoids questions of scheme dependence.

The QCD evolution equations for angular momentum in the flavor singlet case were studied by Ji, Tang and Hoodbhoy [28]. The scheme used corresponds to the choice of a renormalization scheme which preserves chiral symmetry, rather than gauge symmetry [31, 32], so that Σ is scale invariant. The gluon spin then takes the form:

$$\Delta G(t) = -\frac{4\Sigma}{\beta_0} + \frac{t}{t_0}\left(\Delta G(t_0) + \frac{4\Sigma}{\beta_0}\right) , \tag{1}$$

where $t = \ln(Q^2/\Lambda^2_{\rm QCD})$ and $\alpha_s(Q^2) = 4\pi/[\beta_0 \ln(Q^2/\Lambda^2_{\rm QCD}]$, with $\beta_0 = 11 - 2N_f/3$. The total quark and gluon orbital angular momenta satisfy coupled differential equations with solutions which can be written in closed form:

$$\begin{aligned} L^{u+d+s}(t) \ + \ \frac{\Sigma}{2} &= \frac{1}{2}\frac{3N_f}{16+3N_f} + \left(\frac{t}{t_0}\right)^{-\frac{32+6N_f}{9\beta_0}} \left(L^{u+d}(t_0) + \frac{\Sigma}{2} - \frac{3N_f}{16+3N_f}\right) \\ L^g(t) \ &= \ -\Delta G(t) + \frac{1}{2}\frac{16}{16+3N_f} + \left(\frac{t}{t_0}\right)^{-\frac{32+6N_f}{9\beta_0}} \left(L^g(t_0) + \Delta G(t_0) - \frac{1}{2}\frac{16}{16+3N_f}\right) \end{aligned} \tag{2}$$

The solution for the non-singlet case, $L^{u-d} \equiv L^u - L^d$, is much simpler. Note that we now specialize to the case of 3 active flavors ($N_f = 3$):

$$L^{u-d}(t) + \frac{\Delta u - \Delta d}{2} = \left(\frac{t}{t_0}\right)^{-\frac{32}{9\beta_0}} \left(L^{u-d}(t_0) + \frac{\Delta u - \Delta d}{2}\right) . \tag{3}$$

One can also solve for the non-singlet combination $L^{u+d} - 2L^s$ and hence obtain explicit expressions for L^u and L^d (assuming, as in the Myhrer-Thomas work, that $\Delta s = L^s = 0$ at the model scale):

$$\begin{aligned} L^{u(d)} &= -\frac{\Delta u}{2}\left(-\frac{\Delta d}{2}\right) + 0.06 \\ &+ \frac{1}{3}\left(\frac{t}{t_0}\right)^{-\frac{50}{81}}\left[L^{u+d}(t_0) + \frac{\Sigma}{2} - 0.18\right] \\ &+ \frac{1}{6}\left(\frac{t}{t_0}\right)^{-\frac{32}{81}}\left[L^{u+d}(t_0) \pm 3L^{u-d}(t_0) \pm g_A^{(3)} + \frac{\Sigma}{2}\right]. \end{aligned} \tag{4}$$

We are now in a position to evaluate the total and orbital angular momentum carried by each flavor of quark as a function of Q^2, given some choice of initial conditions. Choosing $N_f = 3$, $\Lambda_{\rm QCD} = 0.24$GeV and $Q_0 = 0.4$GeV, together with the values given in Table 1 (and $L(t_0) = \Delta G(t_0) = 0$), we find the results shown in Fig. 1. The behaviour of J^u and J^d is relatively simple, with the former decreasing fairly rapidly at low Q^2 and the latter increasing. Both settle down to slow variation above 1 GeV^2, with the sum around 60% of the total nucleon spin – the rest being carried as orbital angular momentum and spin by the gluons. A similar result has also been reported in the context of the chiral quark soliton model [33].

While the behaviour of $J^{u,d}$ is unremarkable, the corresponding behaviour of $L^{u,d}$ is spectacular. L^u is large and positive and L^d very small and negative at the model scale but they very rapidly cross and settle down inverted above 1 GeV^2 ! The reason for this behaviour is easily understood, because asymptotically L^u and L^d tend to $0.06 - \Delta u/2$ and $0.06 - \Delta d/2$, or -0.36 and +0.28, respectively. This is a model independent result and it is simply a matter of how fast QCD evolution takes one from the familiar physics at the model scale to the asymptotic limit.

As we have already noted, the lattice QCD data for the orbital angular momentum carried by the u and d quarks has a number of systematic errors. Disconnected terms are as yet uncalculated and the data needs to be extrapolated over a large range in both pion mass and momentum transfer in order to extract the physical values of J^u and J^d. Nevertheless, for all these cautionary remarks, the results just reported are consistent with the latest lattice results of Hägler *et al.* [25]. For example, they report J^{u+d} in the range 0.25 to 0.29 at the physical pion mass (their Fig. 47) in comparison with 0.30 in the calculation reported above. They also report $L^{u+d} \sim 0.06$ in comparison with 0.11 in this work. Finally, the qualitative

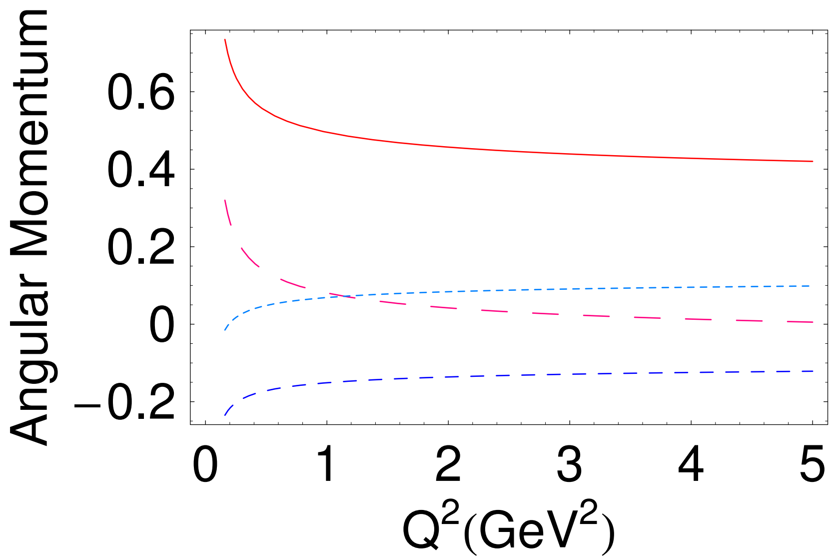

FIG. 1: Evolution of the total angular momentum and the orbital angular momentum of the up and down quarks in the proton - from top to bottom (at 4 GeV^2): J^u (solid), L^d (smallest dashes), L^u (largest dashes) and J^d (middle length dashes). In this case it is assumed that the gluons carry no spin or orbital angular momentum at the model scale (0.4 GeV).

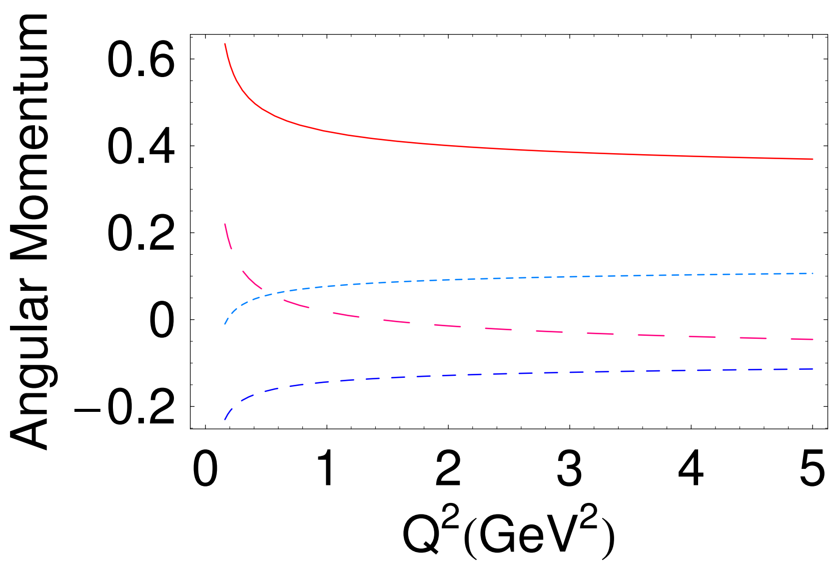

FIG. 2: Evolution of the total angular momentum and the orbital angular momentum of the up and down quarks in the proton - from top to bottom (at 4 GeV^2): J^u (solid), L^d (smallest dashes), L^u (largest dashes) and J^d (middle length dashes). In this case it is assumed that the gluons carry 0.1 units of angular momentum at the model scale (0.4 GeV).

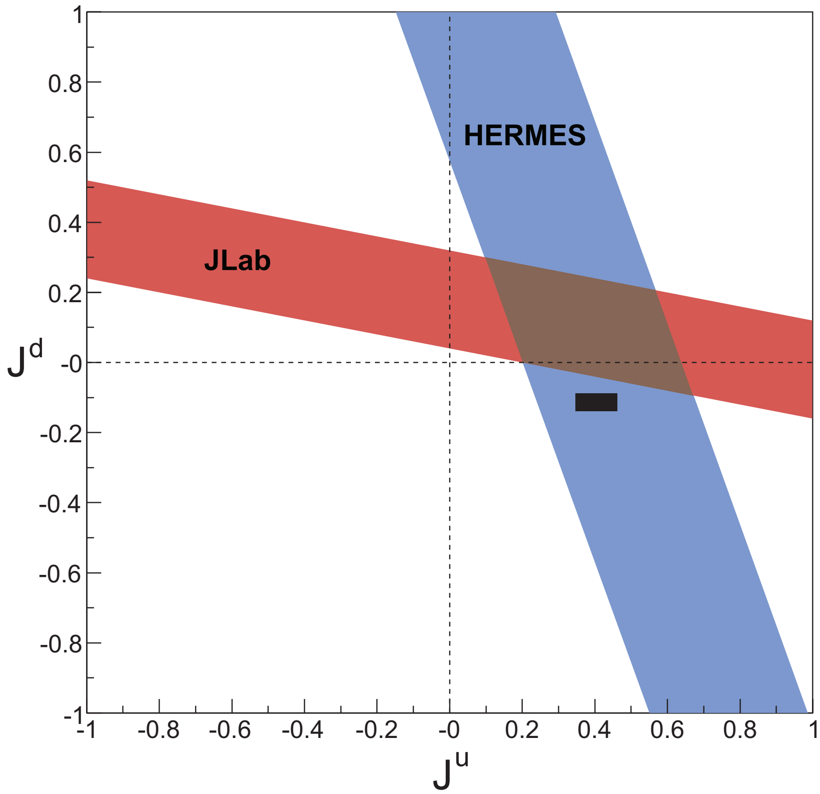

FIG. 3: Comparison between the constraints on the total angular momentum carried by u and d quarks in the proton, derived from experiments on DVCS at Hermes [34, 35] and JLab [36], and the model of Myhrer and Thomas (the small dark rectangle) as explained in this work.

feature that L^d is positive and bigger than L^u is, as we have explained, clearly reproduced in the current work.

Although it is clear that ΔG is too small to give a major correction to the spin sum rule through the axial anomaly [37, 38] (e.g. $-N_f \alpha_s \Delta G/(2\pi) \sim 0.05$ for $\Delta G = 0.3$ at $Q^2 = 3$ GeV^2), it can still be non-zero and it will continue to be critical to pin it down more accurately. As just one example of the effect of a small gluon spin fraction at the model scale, in Fig. 2 we show the evolution of the angular momentum on the u and d quarks if ΔG is set to 0.1 at the starting scale (and $L^{u(d)}$ lowered proportionately to preserve the proton spin). While the qualitative behaviour is identical there are non-trivial quantitative changes. In particular, L^u moves down from 0.01 to -0.03 and J^{u+d} moves down to 0.26 at 4 GeV^2. We note that the nature of the QCD evolution is such that the changes in the values

of L^u and L^d at 4 GeV^2 are considerably smaller than the changes at the model scale. This has the effect of reducing the uncertainty on the predictions of the model at the scale where they can be compared with data.

On the experimental side, the extraction of information about the quark angular momentum is still in its very early stage of development. One needs to rely on a model to analyze the experimental data, which are still at sufficiently low Q^2 that one cannot be sure that the handbag mechanism really dominates. Nevertheless, the combination of DVCS data on the proton from Hermes [34, 35] and the neutron from JLab [36] provides two constraints on J^u and J^d, within the model of Goeke *et al.* [39, 40], as shown in Fig. 3. Also shown there is the prediction of the present work. The uncertainties shown correspond to a few percent variation in the relativistic correction, a 20% reduction in the one-gluon-exchange correction and the uncertainty in the pion cloud correction quoted by Myhrer and Thomas (i.e. $Z - P_{N\pi}/3 + 5P_{\Delta\pi}/3 \in (0.75, 0.80)$). It also includes the variation in the scale between 2 and 4 GeV^2 and the effect of ΔG being as large as 0.1 at the model scale. Clearly, within the present uncertainties, most notably the relatively low Q^2 of the JLab data and the unknown model dependence of the extraction of $J^{u(d)}$, there is a remarkable degree of agreement.

In summary, we have shown that the resolution of the spin crisis proposed by Myhrer and Thomas, which implies that the majority of the spin of the proton resides on u and $\bar{u}$ quarks, after QCD evolution is consistent with current determinations from lattice QCD and experimental data on deeply virtual Compton scattering. The effect of QCD evolution in inverting the orbital angular momentum of the u and d quarks in the model was especially important. For the future, we look forward to improvements in both these areas, with lattice simulations at lower quark mass including the elusive disconnected terms and experimental data at higher Q^2 and x, particularly following the 12 GeV Upgrade at JLab.

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[41] Here Z is the wavefunction renormalization constant, the probability of finding the bare quark structure of the nucleon, and $P_{\mathrm{N}\pi}$ and $P_{\Delta\pi}$ are, respectively, the probabilities of finding a pion with the accompanying baryon having N or Δ quantum numbers.